Out-of-Core Cholesky Factorization Algorithm on GPU and the Intel MIC Co-processors

Ben Chan (Chinese University of Hong Kong)
Nina Qian (Chinese University of Hong Kong)
Mentors: Ed D’Azevedo (ORNL)
Shiquan Su (UTK)
Kwai Wong (UTK)
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Cholesky Factorization

Similar to LU factorization

\[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 5 & 4 \\
0 & 4 & 13
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
1 & 0 & 0 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 0 \\
0 & 4 & 4 \\
0 & 0 & 9
\end{pmatrix}
\]

\[\text{L*U}\]

\[
\begin{pmatrix}
1 & 0 & 0 \\
1 & 2 & 0 \\
0 & 2 & 3
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 0 \\
0 & 2 & 2 \\
0 & 0 & 3
\end{pmatrix}
\]

\[\text{U = L'}\]

- LU works on a wider variety of matrices
- Cholesky factorization on symmetric positive-definite (SPD) matrices only

"Out-of-Core"

- Core – computing unit of the algorithm
  GPU or MIC
  high FLOPS, low storage
- Data stored outside of the core
- Solve larger problems quickly!
Performance Issue with GPU

4 test cases on Keeneland

~160 GFLOPS/PROC
Nvidia M2090 GPU peak performance: 665 GFLOPS

- Performance of algorithm constrained by the hardware architecture
- Quantify data transfer amount
Application: Large scale radiosity problem

• Calculate view factor between any two surfaces in space to form VF matrix (view3d program)
  – Need to separate obstruction-existing case

• Factorize VF matrix by using OOC algorithm
  – Transform VF matrix to SPD first

\[
VF = \begin{pmatrix}
  F_{11} & \cdots & F_{1n} \\
  \vdots & \ddots & \vdots \\
  F_{n1} & \cdots & F_{nn}
\end{pmatrix}
\]

\[
G = \begin{pmatrix}
  \frac{A_1}{\phi_1} - A_1F_{11} & -A_2F_{12} & \cdots & -A_nF_{1n} \\
  -A_1F_{21} & \frac{A_2}{\phi_2} - A_2F_{22} & \cdots & -A_nF_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  -A_1F_{n1} & -A_2F_{n2} & \cdots & \frac{A_n}{\phi_n} - A_nF_{nn}
\end{pmatrix}
\]

Which is a SPD matrix used to solve Boltzmann’s Law
Application: Large scale radiosity problem

• Already done:

• Goal:
  – Improve detecting obstruction part
  – Transfer to Intel MIC co-processor instead of GPU
  – Also need to transfer the OOC algorithm to MIC
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