A MULTI-OBJECTIVE ROBUST STOCHASTIC PROGRAMMING MODEL FOR DISASTER RELIEF LOGISTICS UNDER UNCERTAINTY

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Introduction

Why disaster relief planning?

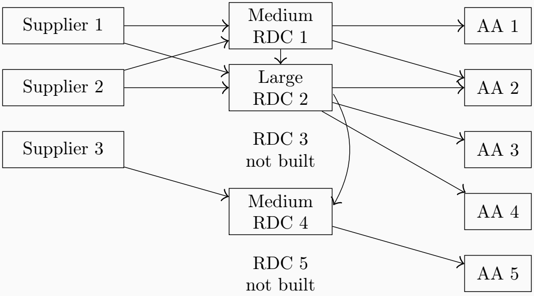
Image Source: DVREE

Introduction

- A "multi-objective robust stochastic programming model" by Bozorgi-Amiri et al. (2013)
  1. To minimize total costs of preparation and reaction measures
  2. To maximize affected areas' overall satisfaction by minimizing the sum of shortage at each affected area
- Project goals:
  1. Improve the aforementioned model in terms of flexibility, speed and solution optimality
  2. Apply the new model to real-life cases
  3. Implement uncertainty quantification (UQ)

Previous model

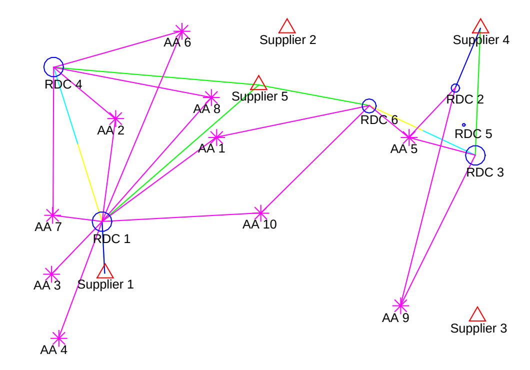
- Suppliers, candidate relief distribution centers (RDCs) and affected areas (AAs):



- Sizes of RDCs, types of commodities and number of scenarios

Current model

- Modifications:
  - Independence of locations of suppliers, RDCs and AAs
  - Mathematical formulation: reducing non-linear constraints to linear constraints



Current model

$$\text{Min Obj}_1 = \text{PRE} + \sum_{s \in S} p_s(\text{POST}_s) + \lambda_1 \sum_{s \in S} p_s \left[\left(\text{POST}_s - \sum_{s' \in S} p_{s'}(\text{POST}_{s'}) \right) + 2\theta_{1s} \right]$$

$$\text{Min Obj}_2 = \sum_{s \in S} p_s \cdot \left(\sum_{c \in C} \max_{k \in K} \{b_{kcs}\} \right) + \lambda_2 \sum_{s \in S} p_s \cdot \left[\left(\sum_{c \in C} \max_{k \in K} \{b_{kcs}\} - \sum_{s' \in S} p_{s'} \cdot \sum_{c \in C} \max_{k \in K} \{b_{kcs'}\} \right) + 2\theta_{2s} \right]$$

such that

Current model

$$\sum_{i \in I} X_{ijcs} + \rho_{jcs} \sum_{i \in I} Q_{ijc} + \sum_{j' \in J \setminus \{j\}} Y_{j'jcs} = \sum_{j' \in J \setminus \{j\}} Y_{jj'cs} + \sum_{k \in K} Z_{jkcs},$$

$$\forall j \in J, c \in C, s \in S$$

$$\sum_{j \in J} Z_{jkcs} - D_{kcs} = I_{kcs} - b_{kcs}, \forall k \in K, c \in C, s \in S$$

$$\sum_{i \in I} \sum_{c \in C} X_{ijcs} \leq M \cdot \sum_{l \in L} \delta_{jl}, \forall j \in J, s \in S$$

$$\sum_{j_2 \in J} \sum_{c \in C} Y_{j_1 j_2 cs} \leq M \cdot \sum_{l \in L} \delta_{j_1 l}, \forall j_1 \in J, s \in S$$

$$\sum_{j_1 \in J} \sum_{c \in C} Y_{j_1 j_2 cs} \leq M \cdot \sum_{l \in L} \delta_{j_2 l}, \forall j_2 \in J, s \in S$$

$$\sum_{k \in K} \sum_{c \in C} Z_{jkcs} \leq M \cdot \sum_{l \in L} \delta_{jl}, \forall j \in J, s \in S$$

Current model

$$\sum_{i\in I}\sum_{c\in C} v_c \cdot Q_{ijc} \leq \sum_{l\in L} \mathrm{Cap}_l \cdot \delta_{jl},\ \forall j \in J$$

$$\sum_{j\in J} Q_{ijc} \leq S_{ic},\ \forall i \in I, c \in C$$

$$\sum_{j\in J} X_{ijcs} \leq \rho_{ics} \cdot S_{ic},\ \forall i \in I, c \in C, s \in S$$

$$\sum_{l\in L} \delta_{jl} \leq 1,\ \forall j \in J$$

$$\mathrm{POST}_s - \sum_{s'\in S} p_{s'}(\mathrm{POST}_{s'}) + \theta_{1s} \geq 0,\ \forall s \in S$$

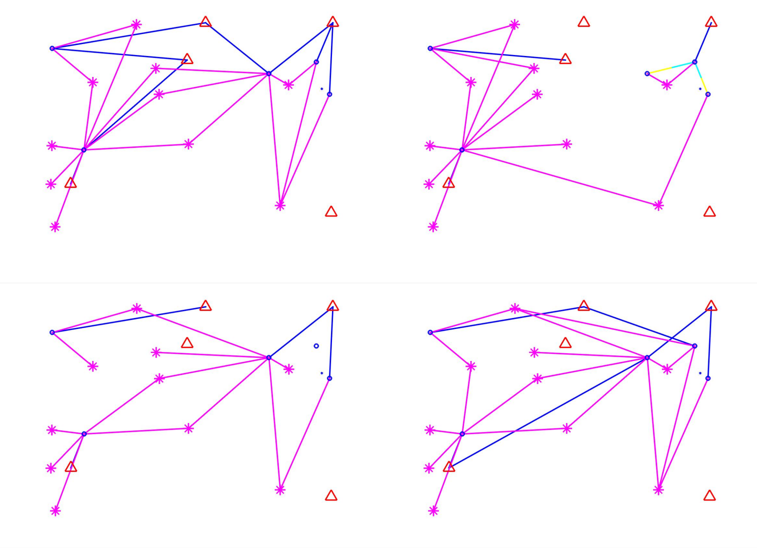
$$\sum_{c\in C} \max_{k\in K}\{b_{kcs}\} - \sum_{s'\in S} p_{s'} \cdot \left(\sum_{c\in C} \max_{k\in K}\{b_{kcs'}\}\right) + \theta_{2s} \geq 0,\ \forall s \in S$$

$$\delta_{jl} \in \{0,1\}, Q_{ijc}, X_{ijcs}, Y_{j_1 j_2 cs}, Z_{jkcs}, I_{kcs}, b_{kcs}, \theta_{1s}, \theta_{2s} \geq 0$$
$$\forall i \in I, j, j_1, j_2 \in J, k \in K, l \in L, c \in C, s \in S$$

Work in Progress

- Write the program for the new model in C using the callable library of SYMPHONY
  - Input all objectives and constraints by specifying the coefficients of each parameter and variable
- Run cases
  - Small case: 5 suppliers, 6 RDCs, 10 AAs, 10 scenarios
  - Medium case: 10 suppliers, 20 RDCs, 80 AAs, 30 scenarios
  - Large case: 50 suppliers, 100 RDCs, 500 AAs, 10 scenarios
  - 3 possible sizes of RDCs and 3 types of commodities in all cases
- Visualize the results using Matlab

Some figures for the small case



Future Work

- Implement parallel computing
- Do real-life cases
  - 500 suppliers, 500 RDCs, 500 AAs and 1000 scenarios
- Explore the possibility of using uncertainty quantification in this model
  - PSUADE

Questions?