High Performance Traffic Assignment Based on Variational Inequality

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Agenda

Introduction

- Traffic Assignment Problem
- Variational Inequality

Progress

Objective

- GPU Implementation
- DTA by dVI

Introduction

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Traffic Assignment Problem

Node

Link

Origin-Destination Pair

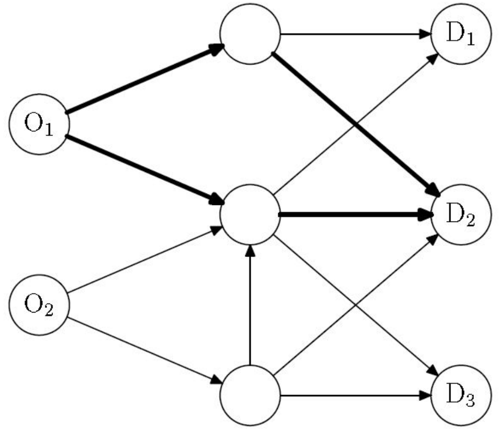
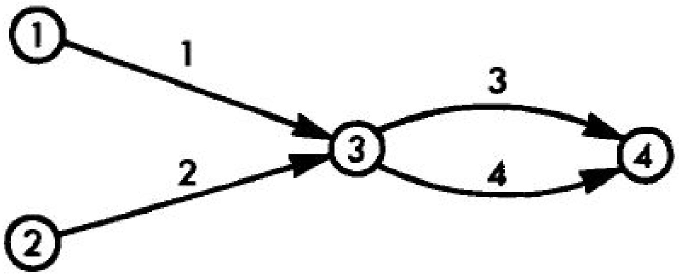


Figure 1.5: An illustration of the traffic equilibrium problem.

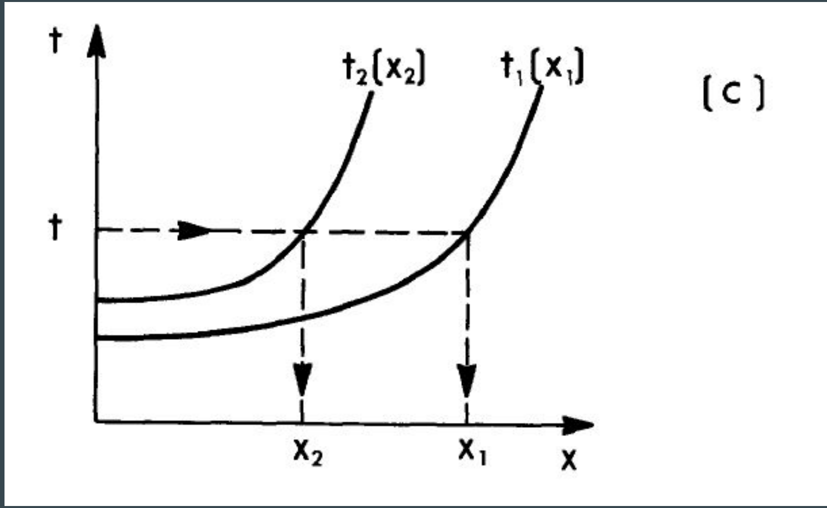


Time Cost

Traffic Assignment Problem

Optimization

- System equilibrium
- User equilibrium



Time Cost Function

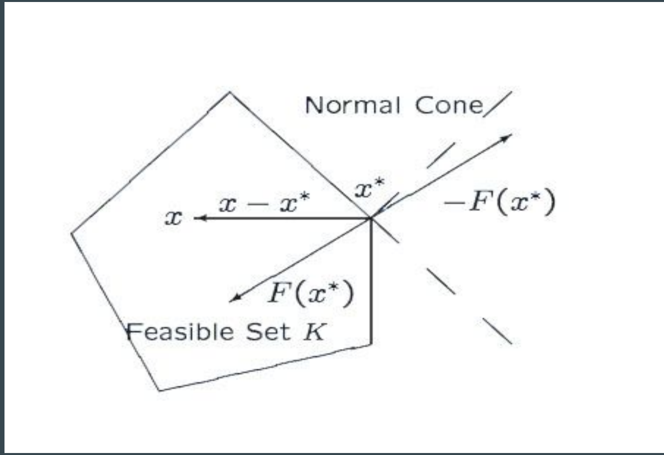
$$t_a = t_a^0(1 + \frac{x_a}{k_a})^4, \forall a \in A.$$

Variational Inequality

- What?
  - definition

$$(y-x)^T F(x) \geq 0, \ \forall y \in K$$

  - Graphically



Variational Inequality

❖ Category

$$
\begin{array}{ccccc}
\text{VI } (K, q, M) & & & & \text{VI } (K, q, M) \\
\Uparrow & & & & \Downarrow \\
\text{VI } (K, F) & \Rightarrow & \text{linearly constrained VI} & \Rightarrow & \text{AVI } (K, q, M) \\
\Downarrow & & \Updownarrow & & \Updownarrow \\
\text{CP } (K, F) & \Rightarrow & \text{MiCP } (F) & \Rightarrow & \text{MLCP} \\
 & & \Downarrow & & \Downarrow \\
 & & \text{NCP } (F) & \Rightarrow & \text{LCP } (q, M).
\end{array}
$$

Variational Inequality

- Why?
  - Intuitive: Either scenorio A or scenorio B
  - closely related to equilibrium
- Application
  - Nash Equilibrium Problem
  - Economic Equilibrium Problem
  - Pricing America Options
  - Frictional Contact Problem
  - Traffic Equilibrium Problem

Progress

Traffic Assignment Problem

- Category
  - Static Traffic Assignment
  - Dynamic Traffic Assignment (continuous or discrete)

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VI on Static Traffic Assignment Problem (STA)

$$\sum_{k \in R_w} f_k^w = q_w,$$

$$C_k^w = \sum_{a \in A} \delta_{ak}^w t_a(x),$$

$$x_a = \sum_{w \in W} \sum_{k \in R_w} \delta_{ak}^w f_k^w,$$

$$u_w \geq 0.$$

Traffic Problem

$$0 \leq f \perp C(\Delta f) - \Lambda^\top u \geq 0$$

$$\Lambda f - q = 0$$

$$u \geq 0$$

$$F(f, u) = \begin{pmatrix} C(\Delta f) - \Lambda^\top u \\ \Lambda f - q \end{pmatrix}$$

Nonlinear complementarity problem

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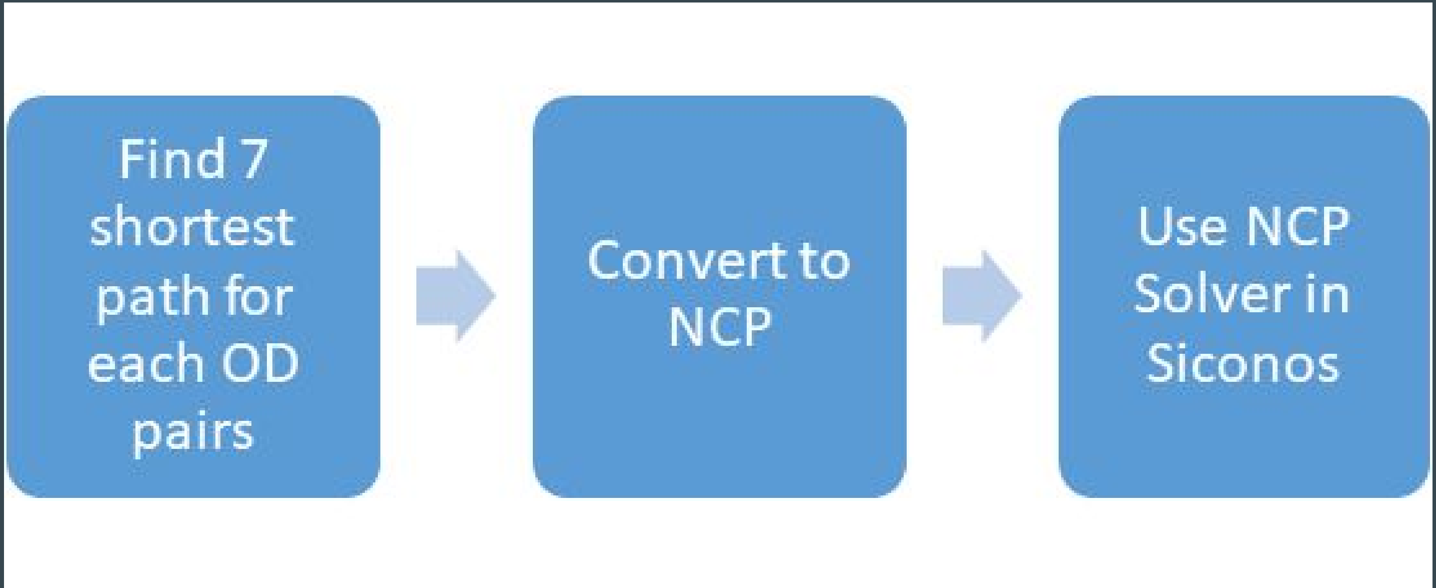
VI on Static Traffic Assignment Problem (STA)

- Limitation
  - Unrealistic to find all path
- Solution
  - Find 7 nonsimilar path for each OD-pair to reduce Matrix size
  - Use Shortest Path Algorethm
  - Get approximate Optimization

Sequential Code



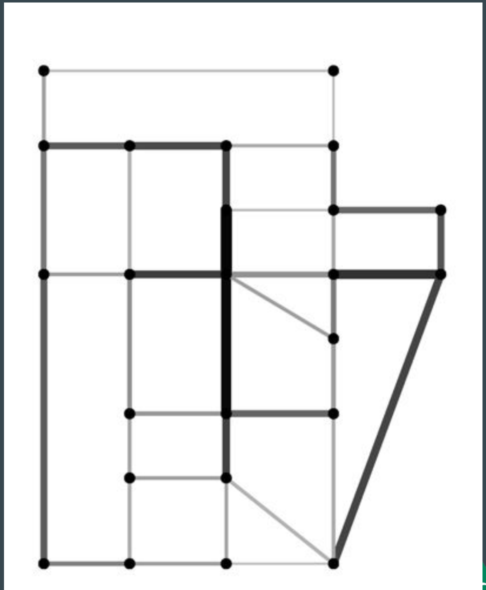
Sequential Code

Sample Input

```
<LINKS>
~ Init node | Term node | Capacity | Length | Free Flow Time | B | Power | Speed limit | Toll | Type
   1     2  25900.200640  6.000000  6.000000  0.150000  4.000000  0.000000  0.000000  1
   1     3  23403.473190  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   2     1  25900.200640  6.000000  6.000000  0.150000  4.000000  0.000000  0.000000  1
   2     6  4958.180928  5.000000  5.000000  0.150000  4.000000  0.000000  0.000000  1
   3     1  23403.473190  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   3     4  17110.523720  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   3    12  23403.473190  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   4     3  17110.523720  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   4     5  17782.794100  2.000000  2.000000  0.150000  4.000000  0.000000  0.000000  1
   4    11  4908.826730  6.000000  6.000000  0.150000  4.000000  0.000000  0.000000  1
   5     4  17782.794100  2.000000  2.000000  0.150000  4.000000  0.000000  0.000000  1
   5     6  4947.995469  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   5     9  10000.000000  5.000000  5.000000  0.150000  4.000000  0.000000  0.000000  1
   6     2  4958.180928  5.000000  5.000000  0.150000  4.000000  0.000000  0.000000  1
   6     5  4947.995469  4.000000  4.000000  0.150000  4.000000  0.000000  0.000000  1
   6     8  4898.587646  2.000000  2.000000  0.150000  4.000000  0.000000  0.000000  1
   7     8  7841.811310  3.000000  3.000000  0.150000  4.000000  0.000000  0.000000  1
   7    18  23403.473190  2.000000  2.000000  0.150000  4.000000  0.000000  0.000000  1
   8     6  4898.587646  2.000000  2.000000  0.150000  4.000000  0.000000  0.000000  1
   8     7  7841.811310  3.000000  3.000000  0.150000  4.000000  0.000000  0.000000  1
   8     9  5050.193156  10.000000  10.000000  0.150000  4.000000  0.000000  0.000000  1
   8    16  5045.822583  5.000000  5.000000  0.150000  4.000000  0.000000  0.000000  1
```

```
Origin  1
    1 :      0.0;     2 :    100.0;     3 :    100.0;     4 :    500.0;     5 :    200.0;
    6 :    300.0;     7 :    500.0;     8 :    800.0;     9 :    500.0;    10 :   1300.0;
   11 :    500.0;    12 :    200.0;    13 :    500.0;    14 :    300.0;    15 :    500.0;
   16 :    500.0;    17 :    400.0;    18 :    100.0;    19 :    300.0;    20 :    300.0;
   21 :    100.0;    22 :    400.0;    23 :    300.0;    24 :    100.0;
```

Output



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VI on Dynamic Traffic Assignment Problem (DTA)

Solve for dynamic cost function

$$\frac{dx^p_{a_1}(t)}{dt} = h^{\tau,k}_p(t) - g^p_{a_1}(t) \quad \forall p \in \mathcal{P}$$

$$\frac{dx^p_{a_i}(t)}{dt} = g^p_{a_{i-1}}(t) - g^p_{a_i}(t) \quad \forall p \in \mathcal{P}, i \in [2, m(p)]$$

$$\frac{dg^p_{a_i}(t)}{dt} = r^p_{a_i}(t) \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$\frac{dr^p_{a_1}(t)}{dt} = R^p_{a_1}(x, g, r, h^{\tau,k}) \quad \forall p \in \mathcal{P}$$

$$\frac{dr^p_{a_i}(t)}{dt} = R^p_{a_i}(x, g, r) \quad \forall p \in \mathcal{P}, i \in [2, m(p)]$$

$$x^p_{a_i}((\tau - 1) \cdot \Delta) = x^{p,0}_{a_i} \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$g^p_{a_i}((\tau - 1) \cdot \Delta) = 0 \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$r^p_{a_i}((\tau - 1) \cdot \Delta) = 0 \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

Solve dVI

$$\text{find } h^* \in \Lambda \text{ such that}$$

$$\sum_{p \in \mathcal{P}} \int_{t_0}^{t_f} \Psi_p(t, h^*)\left(h_p - h^*_p\right) dt \geqslant 0$$

$$\forall h \in \Lambda$$

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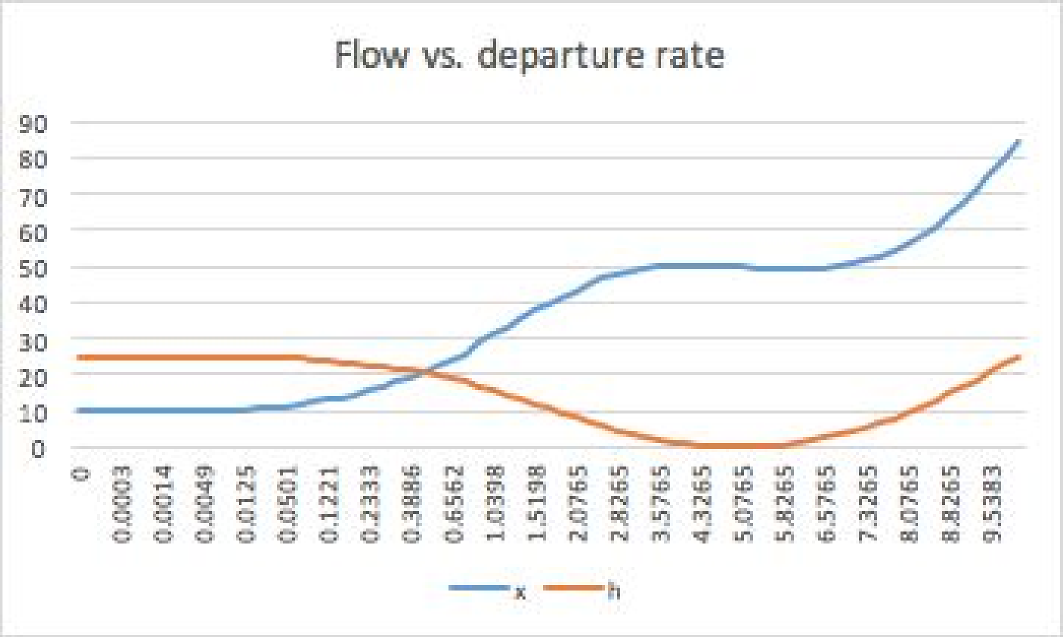
DTA Cost function



Objective

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GPU Implementation

- In Shortest Path Algorithm
  - Use GPU to Inplement
- In NCP Solver
  - Use GPU to direct calculate Sparse Matrix

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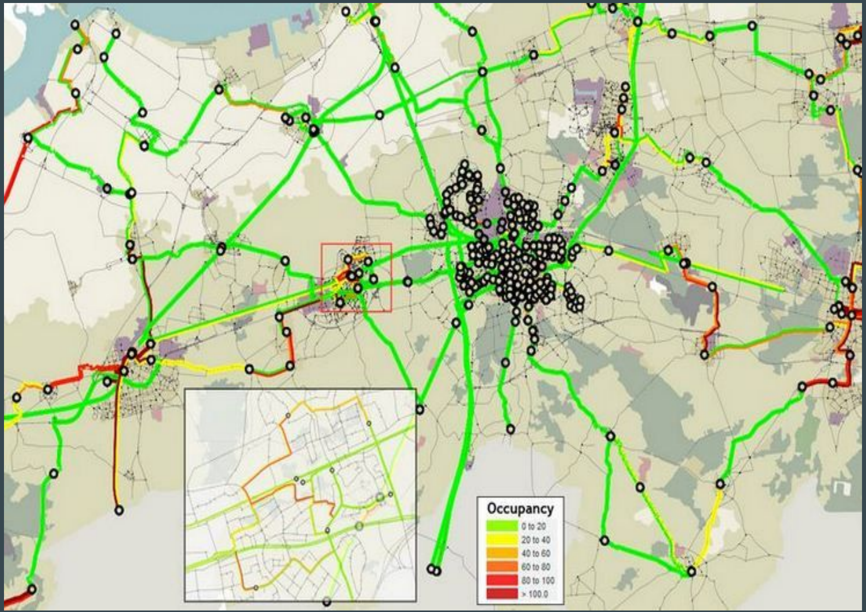
DTA(dynamic traffic assignment)

$$\text{find } h^* \in \Lambda \text{ such that}$$

$$\sum_{p \in \mathcal{P}} \int_{t_0}^{t_f} \Psi_p(t, h^*)\left(h_p - h_p^*\right) dt \geqslant 0$$

$$\forall h \in \Lambda$$



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END

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